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OPTIMIZATION OF THE ROBUST STABILITY LIMIT FOR MULTI-CUTTER TURNING PROCESSES

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ABSTRACT

Multi-cutter turning systems bear huge potential in increasing cutting performance. In this study we show that the stable parameter region can be extended by the optimal tuning of system parameters. The optimal parameter regions can be identified by means of stability charts. Since the stability boundaries are highly sensitive to the dynamical parameters of the machine tool, the reliable exploitation of the so-called stability pockets is limited. Still, the lower envelope of the stability lobes is an appropriate upper boundary function for optimization purposes with an objective function taken for maximal material removal rates. This lower envelope is computed by the Robust Stability Computation method presented in the paper. It is shown in this study, that according to theoretical results obtained for optimally tuned cutters, the safe stable machining parameter region can significantly be extended, which has also been validated by machining tests.

INTRODUCTION

Multi-cutter turning heads are used in industry for producing cylindrical parts in order to increase machining productivity via high material removal rate (MRR). Nevertheless, high accuracy can only be achieved, if the cutting process is stable, namely, if it is free of harmful chatter vibrations [1, 2, 3, 4].

Stability of machining operations is usually represented on so-called lobe-diagrams in the plane spanned by the parameters w as width of cut and Ω as spindle speed of the workpiece [5, 6]. Typically, the boundary curves divide the parameter plane into stable (below curve) and unstable (above curve) regions.

The continuous increase of productivity is the major objective function set by industry. Thus, in order to assure high material removal rates [7], the regions in the vicinity of the intersection points of two adjacent lobes, the so-called stability pockets are usually exploited for machining. However, difficulties occur if the parameters, the stability computations are based on, are subjected to uncertainties, due to the high sensitivity of the stability boundary curves to the dynamical parameters of the machine tool. The vertical asymptotes of the stability lobes, for example, are strongly influenced by the precise values of the natural frequencies of the cutters, thus, the horizontal position of the lobes determining the location of the pockets along the spindle speed parameter is reliable in a limited way only.

Nevertheless, it is an essential requirement to avoid unstable parameter regions, so a more promising solution is provided by selecting machining points far off the boundary of stability not even risking the emergence of harmful chatter vibrations during machining. In order to assure high MRR values and stable machining at the same time, stability boundaries have to be shifted up together with their lower envelope as much as practically possible. Among other essential methods, optimal tuning of system parameters can lead to significant extension of the safe stable parameter region. Some of the corresponding theoretical background was already shown in [8].

In this study the Robust Stability Computation Method is applied for the computation of the aforementioned lower envelope curve. Stability computation for the applied mechanical model predicted an optimal value for the stiffness ratio of two turning tools. Experimental validation of theoretical results is given in this paper, too.

STABILITY OF 2-CUTTER TURNING OPERATIONS

In the applied mechanical model (see Fig.1), for sake of simplicity, we consider the tools to be flexible only in feed direction, which implies that the equations of motion are delay-differential equations. This can be derived from the phenomenon of the well-known surface regeneration effect with certain modifications. This phenomenon originates in the fact, that the chip thickness is influenced by the instantaneous position of the tool and also by the position of the other tool half a revolution before. The single cutter regenerative model for orthogonal turning with one degree of freedom can be found for example in [9], from which the model of a multi-cutter turning system can be derived (see details in [8]).

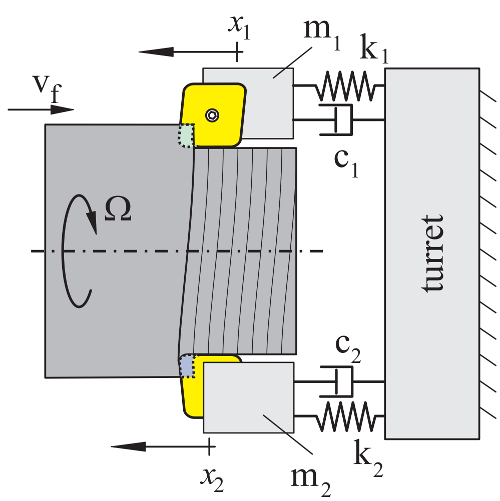

FIGURE 1. MECHANICAL MODEL OF THE 2-CUTTER SYSTEM.

The equation of motion for a 2-cutter case has the following form:

$$\mathbf{M}\begin{bmatrix}\ddot{x}_1(t)\\ \ddot{x}_2(t)\end{bmatrix}+\mathbf{C}\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix}+\mathbf{K}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}= k_w\left(\begin{bmatrix}v_f\tau\\ v_f\tau\end{bmatrix}+\begin{bmatrix}x_2(t-\tau)\\ x_1(t-\tau)\end{bmatrix}-\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}\right) \qquad (1)$$

where $\mathbf{x}=[x_1\ x_2]^T$ is the vector of the coordinates representing the motion of the cutters in feed direction, $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ is the damping matrix, $\mathbf{K}$ is the stiffness matrix. In the right hand side, $k_w = k_f w$ is the so-called cutting coefficient, k_f is the derivative of the cutting force with respect to the chip thickness in case of unit chip width w and stationary cutting parameters. The static component of the cutting force is determined by the feed velocity v_f. The time-delay τ is inversely proportional to the spindle speed Ω. In the chosen mechanical model, the cutters are considered to be dynamically decoupled, so the matrices $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ are diagonal. The diagonal elements of the damping matrix $\mathbf{C}$ are $c_i = 2\zeta_i\sqrt{m_i k_i}$ belonging to the corresponding tools ($i=1,2$), where ζ_i is the damping ratio.

Let us assume the vector of general coordinates in the following form: $\mathbf{x}(t)=\mathbf{x}_0+\mathbf{u}(t)$, where $\mathbf{x}_0$ is the stationary solution of the differential Eq.(1) and $\mathbf{u}(t)$ is the small perturbation around the stationary state. After substituting the trial solution into the perturbed form of Eq.(1) in the exponential form $\mathbf{u}(t)=\mathbf{A}e^{-\lambda t}$ with complex coefficient vector $\mathbf{A}$ and characteristic exponent λ, one obtains the characteristic equation as a determinant:

$$D(\lambda)=\det\left(\mathbf{M}\lambda^2+\mathbf{C}\lambda+\mathbf{K}+k_w\mathbf{I}-k_w\begin{bmatrix}0 & e^{-\lambda\tau}\\ e^{-\lambda\tau} & 0\end{bmatrix}\right)=0, \qquad (2)$$

where $\mathbf{I}$ is the unit matrix. On the stability boundary, the characteristic exponents are pure imaginary: $\lambda = \mathrm{i}\omega_c$, where ω_c is the chatter frequency.

STABILITY AND ROBUST STABILITY

Stability boundaries are computed based on the D-subdivision method, where the real and imaginary parts of the characteristic equation are analysed [9]. Parameters Ω and w of the stability chart and the unknown chatter frequency ω_c form the parameter set of the following system of non-linear equations:

$$\mathrm{Re}(D(w,\Omega,\omega_c)) = 0\,, \qquad (3)$$

$$\mathrm{Im}(D(w,\Omega,\omega_c)) = 0\,. \qquad (4)$$

Eq.(3) and (4) represent a co-dimension 2 problem in the space of 3 parameters, which can be solved efficiently with the Multi-Dimensional Bisection Method (MDBM). This is a fast algorithm able to find all the roots automatically [10, 11]. The MDBM method provides the gradients of Eq.(3) and (4) along the boundary curves, which allows the computation of the so-called instability gradients [12]. This shows, which region along the boundary curves has a higher number of unstable characteristic exponents. Trivially, the points belonging to width of cut value $w = 0$ are stable, thus one can deduce the stable and unstable regions.

The resulting stability boundaries are presented by black lines and the stable regions are shaded gray in Fig.2 for a 2-cutter system with different dynamic parameters for each cutter, based on the idealized system parameters of the experimental fixture presented in the subsequent section.

The D-subdivision method requires the exact model of the mechanical system and does not take into account the uncertainty of the input parameters. Inaccurate natural frequency and spindle speed can endanger safe machining in the stability pockets, since they severely influence the horizontal position of the machining parameter points relative to the lobe curves. To eliminate the risk of sliding into an unstable machining parameter domain, in industrial applications, one has to be aware of the lower envelope of the stability boundaries, which can, for instance, be computed by the Robust Stability Computation Method [13]. The main steps of this method are summarized below.

A new additional parameter is introduced, $\phi := \tau\omega_c$, the so-called regenerative phase shift. This defines the phase shift between the current position $x(t)$ and the delayed position $x(t-\tau)$. This parameter ϕ can be used to characterize the uncertainty in the time delay parameter caused by the fluctuations in the spindle speed. It is considered to be an independent extra parameter in the exponential term in Eq.(3),(4):

$$D(w,\Omega,\omega_c,\phi) = \det\left(\mathbf{M}\lambda^2 + \mathbf{C}\lambda + \mathbf{K} + k_w\mathbf{I} - k_w \begin{bmatrix} 0 & e^{-i\phi} \\ e^{-i\phi} & 0 \end{bmatrix}\right) = 0\,. \quad (5)$$

The resultant co-dimension 2 problem in the extended 4 dimensional parameter space defines a surface. The robust stability limit is determined by the envelope of this surface, where the normal vectors of the surface segments are perpendicular to the ω_c axis. It was found, that in the vicinity of these parameter points, the real parts of the roots λ of the characteristic equation Eq.(5) do not change as a function of the perturbation parameter (for details see [13]). This condition can be described as follows:

$$\mathrm{Re}\left(\frac{\partial\lambda}{\partial\phi}\right) = 0\,. \quad (6)$$

Based on the implicit derivation of Eq.(5) (see in [9]), a straightforward calculation leads to

$$\mathrm{Im}\left(\overline{\frac{\partial D(\omega_c)}{\partial\omega_c}}\,\frac{\partial D(\omega_c)}{\partial\phi}\right) = 0\,, \quad (7)$$

which defines the robust stability limit.

The MDBM is an appropriate tool to solve the resultant co-dimension 3 problem in the extended 4 dimensional parameter space $(\Omega, w, \omega_c, \phi)$, because its application is automatic and more efficient than the usage of other existing methods like continuation or brute force methods. Due to high computation efficiency of the MDBM [10], the robust stability limit is determined within a few seconds.

The resultant robust stability limits are presented with blue lines in Fig.2. Multiple solutions for the above system of equations exist, but the robust stability limit is only represented by the lowest envelope curve. It can furthermore be seen, that the robust stability limit is independent of the spindle speed. Note, that this property is true only in the special case when process damping is not taken into consideration in the model. However, the Robust Stability Computation Method works for envelope curves of arbitrary shape, too.

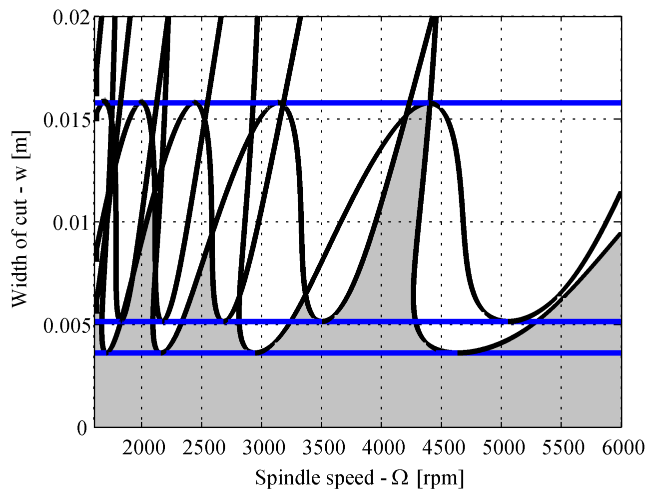

FIGURE 2. STABILITY CHART AND THE ROBUST STABILITY LIMIT OF THE DETUNED SYSTEM WITH NORMALIZED PARAMETERS: $m_1 = m_2 = 11$ kg, $k_1 = 14.5$ N/μm, $k_2 = 7.032$ N/μm, $\zeta_1 = \zeta_2 = 0.66$ %, $k_\mathrm{f} = 237$ N/mm^2

OPTIMAL DETUNING

Our main goal is to find the optimal stiffness parameters of the multi-cutter turning system for which the safe stable parameter region is maximal. Our practical experience shows that one can adjust the stiffness of the cutters with simple engineering solutions, while the other dynamical parameter values are not affected much. In this respect, the stiffness ratio $\beta := k_2/k_1$ is a relevant parameter which is to be analyzed. Only $k_2 < k_1$ is of interest, since we assume the machine tool to be built with the maximal achievable stiffness and in practice we can only decrease the stiffness value. For this optimization procedure, the relative damping ratio is assumed to be unchanged. Robust stability limits for different β values are presented in Fig.3.

It can be proved, that for a 2-cutter turning system where $k_1 = k_2$, the robust stability limit remains the same as for the 1-cutter case, its analytic formula is given by $w_{\mathrm{RS}} = 2\zeta(1+\zeta)k_1/k_{\mathrm{f}}$ (see [9]); with parameters used in Fig.3, this gives w_{RS} = 0.813 mm. In the same figure it is visible that there exists an optimal value for the stiffness ratio, where the robust stability boundary is maximal. For the given dynamical parameters, this optimal value can be obtained from Fig.3: $\beta_{opt} = k_2/k_1$ = 0.485, where w_{RS} = 3.63 mm. This means an increase of the robustly stable width of cut limit of 447 %.

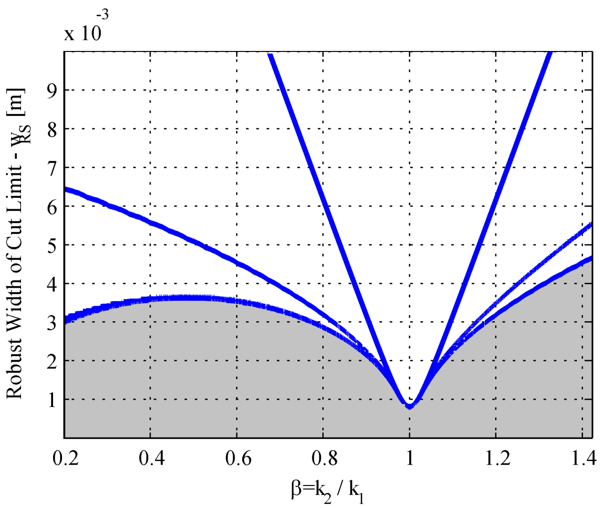

FIGURE 3. ROBUST STABILITY LIMIT OF DETUNED SYSTEM AS A FUNCTION OF STIFFNESS RATIO WITH PARAMETERS: $m_1 = m_2 = 11$ kg, $k_2 = 14.5$ N/μm, $\zeta_1 = \zeta_2 = 0.66$ %, $k_{\mathrm{f}} = 237$ N/mm^2

Experimental validation

A 2-cutter fixture structure was designed and built for the validation of the theoretical results, see Fig.4. The structure was symmetric and could be detuned for the stiffness parameters. The corresponding dynamical parameters of the system were identified by standard modal testing methods [14, 15]. Cutting tests were performed in order to monitor stable and unstable machining points. The chatter vibration was detected by piezo-accelerometers placed on the different branches of the fixture close to the tool tips [16]. Measurement points are presented for the symmetric case in Fig.5 and for the detuned case in Fig.6. Unstable points are denoted with red crosses, stable points with green circles and marginal points with purple diamonds. For the symmetric two-cutter system with fitted dynamic parameters obtained by modal testing, the computed theoretical robust stability limit is $w_{RS} = 0.8$ mm and for the detuned two-cutter system it is $w_{RS} = 3.6$ mm, which is an improvement in stability of a factor of 4. In practice, it is nearly impossible to create a fixture for two cutters, which is able to eliminate physical coupling. Although this physical coupling may require improvement in the modelling, still, the measurement points and the theoretical results show good correlation.

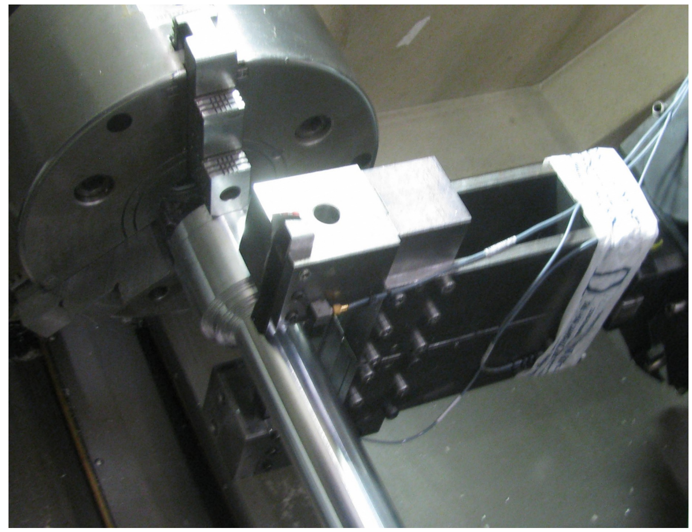

FIGURE 4. TWO-CUTTER TEST FIXTURE

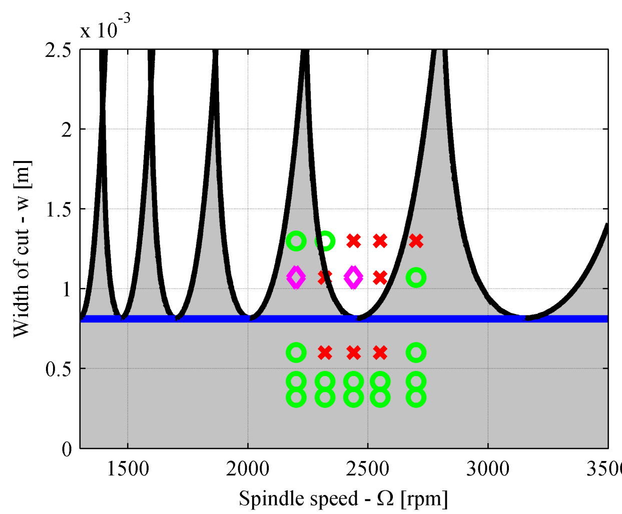

FIGURE 5. STABILITY CHART, MEASUREMENT POINTS AND THE ROBUST STABILITY LIMIT OF THE SYMMETRIC SYSTEM WITH PARAMETERS: $m_1 = 10.778$ kg, $m_2 = 11.118$ kg, $k_1 = 14.2$ N/μm, $k_2 = 14.7$ N/μm, $\zeta_1 = 0.65$ %, $\zeta_2 = 0.67$ %, $k_{\mathrm{f}} = 237$ N/mm^2

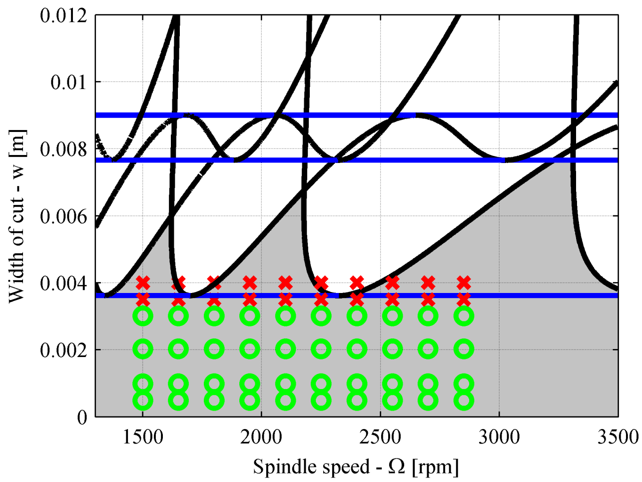

FIGURE 6. STABILITY CHART, MEASUREMENT POINTS AND THE ROBUST STABILITY LIMIT OF THE DETUNED SYSTEM WITH PARAMETERS: $m_1 = 5.2$ kg, $m_2 = 8.57$ kg, $k_1 = 4.11$ N/μm, $k_2 = 2.6$ N/μm, $\zeta_1 = 6.86$ %, $\zeta_2 = 4.09$ %, $k_{\mathrm{f}} = 237$ N/mm^2

CONCLUSION

The safe stability limit of a 2-cutter turning process is represented by the maximal chip width value w_{RS}, below which the machining is stable for all spindle speeds. The objective of this study was to optimize the process with respect to the maximal robustly stable parameter domain. The safe stability limit for symmetrical and detuned 2-cutter turning systems was computed by means of a Robust Stability Computation Method. It was shown, that for practically relevant applications, there exists an optimal stiffness ratio β_{opt} for the detuned system, which was calculated. Cutting tests were carried out, the results of which validated the increase of the safe stability limits for the multi-cutter turning process.

ACKNOWLEDGMENT

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